DECENTRALIZED APPROACH FOR POWER SYSTEM STABILITY ANALYSIS FOLLOWING LIGHTNING EVENTS

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ABSTRACT

The stability analysis of multimachine power systems following lightning disturbances involves studies of large sets of non-linear differential equations. A new approach to quantitatively study a large power system's transient stability is presented. The n-machine power system can be decomposed into $\frac{(n-1)n}{2}$ subsystems. From the variation of total system energy following lightning disturbances (equal area criteria), a set of the critical clearing angles (phase angles), one for each machine pair, can be calculated. If the initial conditions of the power system following a lightning disturbance are within the critical clearing region at the time of return to normal operation, the system will be stable. If the angular difference between any pair of machines exceeds the calculated respective critical clearing angle, then the system is said to be unstable.

1. INTRODUCTION

In any large power system, stability is maintained so long as the motors and generators in the system are in synchronous operation (1). In order to understand all aspects of the power system when it is subjected to an unusual disturbance such as a lightning strike somewhere in the system, a large number of components and phenomena must be studied which cover a considerable spectrum of response time. Basic models for power system dynamics have previously been evaluated by many investigators (2, 3) as they applied to various off-line and on-line conditions. This paper presents a new approach to evaluating power systems influenced by lightning events.

1.1 Types of Stability Problems

In order to understand the technique involved, the operational modes must first be examined. There are four types of stability modes, each with its associated stability problem. The first type, the steady state (S.S.) exists if the power system is stable when not subject to aperiodic disturbances.

The second type of stability is applicable to the lightning problem. Transient stability is implied if the power system operates in a stable condition when a sudden change of system structure, such as a suddenly applied overload or fault condition, occurs. We assume some system components like Automatic Voltage Regulators (AVR) and Governor-Turbines are immune to stability problems associated with lightning transients.

In transient stability analysis, the speed ω is assumed to be constant while the angular position with respect to the synchronous reference frame is considered to be a function of time. Therefore, the model presented here will be valid only for the first one to two seconds following a disturbance.

The third type is the "Extended Transient Stability" problem. This type extends the transient stability problem by considering the power system in more detail over a longer time period. The model is valid for up to ten seconds following a disturbance.

The fourth type is the long term stability problem. In this type, all the components of the power system are represented in detail. The ω and δ are functions of time.

1.2 Existing Approaches

Steady state stability can currently be analyzed using the eigenvalue method. If the real part of all system eigenvalues are negative, then the system is said to be steady state stable, otherwise the power system is unstable.

Transient stability can be studied using energy balance methods which deal with the limitations of system energy. If the system energy following a transient condition exceeds a certain quantity, machines in the system will be "broken" away from each other; this indicates an instability. When a stability problem involves AVR and Governor-Turbine machines, other types of stability methods are used to model the power system components.

As will be seen, the analysis of power system stability following transient disturbances such as a lightning event will involve the study of a large set of non-linear differential equations for even a small size system. Today's power systems are growing to massive proportions due to increased population and industrial development. The increased quantities of transmission lines and machines, plus the increasing need for power from existing transmission lines, has resulted in increasingly complex stability maintenance requirements.

The following sections explain the existing methods that have been developed in order to study the transient stability of power systems. Not all the methods previously developed are useful when examining the fast rise-time transient associated with a lightning event.

1.3 Simulation Method

In this method, machine stability information can be obtained by actually integrating the machine voltage swing equations. If any one machine or any group of machines swing away from the rest of the machines in the system then the system is said to be unstable, otherwise the system is stable. The major disadvantage of this method for lightning applications is that it is necessary to observe the system behaviour for a relatively long time (e.g. two or three seconds) before one can conclude whether the system is going to be stable or not. In addition, the method is inefficient since the system may be unable to restore its normal operation after the instability is detected.

1.4 Direct Method

More recently, the second also known as the direct method of Lyapunov has been applied to transient stability problems involving power systems and many papers (4)(5)(6) (7)(8) have been published in this area. But the direct method of Lyapunov is severely limited when applied to problems of high dimension systems, (i.e. complication of calculations increases as the number of machines of the power system are increased). Since modern systems have become extremely large and complex, the stability information of a particular machine affected by a system fault will be virtually buried in the mass of calculations.

1.5 Decomposition Approach

For reasons previously mentioned, it may be advantageous to view the high order systems as being composed of several lower order sub-systems, which when interconnected in an appropriate fashion, yield the original composite or interconnected system. The stability analysis of such systems can then often be accomplished in terms of the simpler subsystems, and the interconnecting structure of such a composite system. In this way, it is possible to circumvent complications which usually arise when the direct method is applied to high order systems.

Jocic, Pavalla and Siljak (9) investigated the idea of "Vector Lyapunov Functions." This approach developed the stability criteria for individual machines when disconnected from the system, while ignoring the interactions between machines. After the individual stability is established, the stability criteria for all the system interconnections are established. As a conclusion, if the power system satisfies both stability criteria for individual machines and system interconnections, then the power system is stable. This approach has ignored one of the most important features of power systems which states that unstable machines may be stabilized by the interactions with other machines in the system. Therefore, this method is considered very conservative.

1.6 Component Connection Approach

The component connection approach (10) was used to study the stability of power systems via a different approach from other methods.

(1) A component connection model is used to represent the large scale dynamic system (LSDS).

(2) Each single input and single output dynamic component is replaced by two constant parameters in the considerations of the stability problems. This will reduce the complexity of the stability problems.

(3) The objective of the stability problem is to find all positive limiting sets including the case of asymptotic stability, with $x = 0$ as a special case. (x is the system status.)

(4) It gives a condition for multi-swing stability by placing conditions in each line separately.

When a positive limiting set consists of a-singlepoint $x = 0$, we then have the asymptotic stability of $x = 0$. The positive limiting set is the set which other solutions approach as time tends to infinity.

The component connection model is useful because the component equations and connections are explicitly expressed in the model so that one can see more clearly how the components and their inter-connectivity enter the problem.

The drawbacks of this approach are listed as follows:

(1) It uses the Lyapunov stability concept, therefore, the conservativeness of Lyapunov's theory enters the approach.

(2) It derives a stability criteria by assuming identical linearized transmission lines, and generalizes the result to the non-linear transmission line system.

(3) The example used in this paper assumes all machines to be identical, otherwise, the formulations of machine models will be difficult. The stability criteria of a linear transmission line is also very complicated.

This approach may present the conservative nature of Lyapunov's second method, and the complexity of finding the stability criteria. However, the approach presented by Lu, Liu, and Jenkins contains a very important idea which motivated the work presented in this paper.

1.7 Decomposition/Equal Area Method

From the above discussion, one can see that a more efficient technique which utilizes the idea of decomposition and aggregation approach should be developed, which will be more applicable to the analysis of large power systems following transient events. This paper decomposes the n-machine power system into a set of $(n^2-n)/2$ equivalent subsystems. Each subsystem represents a simple system of one machine and an infinite bus as shown in Figure 1. The Equal Area criteria method can be used to calculate the critical clearing machine angles. The faulted system will then be simulated with lightning transients. This allows the machine angles to be compared with the previously calculated subsystem machine critical clearing angles. If any critical clearing angle is exceeded by its respective machine angle, then the system is unstable. This method is also useful as an on-line contingency detector. The most elegant contribution from this method is that it permits the use of a local monitoring system due to the fact that the system is decoupled. The use of other optimal stability criteria in the mathematical sense may not be practical and useful for large scale dynamic systems. It usually requires a global monitoring system which may be impractical and expensive. For this reason the approach presented is worthwhile.

The advantages of this approach are as follows:

(1) As discussed above, only the local monitoring system, not the global monitoring system is required to determine the power system stability.

(2) The analysis of this approach can easily include the effect of transfer and load conductances, which cannot easily be included using the direct approach.

(3) The approach will show exactly how the system breaks up. It is not possible for the direct method to show this.

(4) The method is powerful enough to be used to examine virtually instantaneous transient effects.

2. APPROACH

2.1 Mathematical Model

The electrical power generated from each generator within the n-machine power system can be expressed.

$$P_{ei} = Re[\tilde{V}_i \tilde{I}_i^*] = Re[\tilde{V}_i (\sum_{j=1}^{n} \tilde{Y}_{ij}^* \tilde{V}_j^*)] = Re[V_i \angle \delta_i (\sum_{j=1}^{n} Y_{ij} \angle -\theta_{ij} \; V_j \angle -\delta_j)]$$

$$= V_i^2 Y_{ii} \cos \theta_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} V_i V_j Y_{ij} \cos(\delta_{ij} - \theta_{ij}) \quad ; \quad i = 1, \cdots n$$

Substitute Equation(1) into the equation of motion (Equation(2)) of the ith machine.

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = P_{mi} - P_{ei} = [P_{mi} - V_i^2 Y_{ii} \cos \theta_{ii}] - \sum_{\substack{j=1 \\ j \neq i}}^{n} V_i V_j Y_{ij} \cos(\delta_{ij} - \theta_{ij})$$

For simplicity, the machine damping term is assumed to be negligible. Let

$$P_i \triangleq P_{mi} - V_i^2 Y_{ii} \cos\theta_{ii} = \sum_{\substack{j=1\\ j\neq i}}^{n} P_{ij} \quad \text{and} \quad M_i \triangleq \sum_{\substack{j=1\\ j\neq i}}^{n} M_{ij} \tag{3}$$

Equation(2) can now be written as

$$\Big(\sum_{\substack{j=1\\ j\neq i}}^{n} M_{ij}\Big) \frac{d^2\delta_i}{dt^2} = P_i - \sum_{\substack{j=1\\ j\neq i}}^{n} V_i V_j Y_{ij} \cos(\delta_{ij}-\theta_{ij}) = \sum_{\substack{j=1\\ j\neq i}}^{n} \Big[P_{ij} - V_i V_j Y_{ij} \cos(\delta_{ij}-\theta_{ij})\Big] \tag{4}$$

From equation(4), it is realized that for any path (i,j), there are two describing equations,

$$\begin{aligned} \frac{d^2\delta_i}{dt^2} &= \frac{P_{ij}}{M_{ij}} - \frac{V_i V_j Y_{ij} \cos(\delta_{ij}-\theta_{ij})}{M_{ij}} \\ \frac{d^2\delta_j}{dt^2} &= \frac{P_{ji}}{M_{ji}} - \frac{V_j V_i Y_{ji} \cos(\delta_{ji}-\theta_{ji})}{M_{ji}} \end{aligned} \tag{5}$$

By combining the above two equations, equation(6) is formed and used to describe the path (i,j),

$$\frac{M_{ij}M_{ji}}{M_{ij}+M_{ji}} \frac{d^2\delta_{ij}}{dt^2} = \frac{P_{ij}M_{ji} - P_{ji}M_{ij}}{M_{ij}+M_{ji}} - \frac{M_{ji}V_iV_jY_{ij}\cos(\delta_{ij}-\theta_{ij}) - M_{ij}V_jV_iY_{ji}\cos(\delta_{ji}-\theta_{ji})}{M_{ij}+M_{ji}} \tag{6}$$

$$\text{Note:} \quad \tilde{Y}_{ij} = \tilde{Y}_{ji}\,; \quad \theta_{ij} = \theta_{ji}\,; \quad \delta_{ij}+\theta_{ij} \neq -\delta_{ij}+\theta_{ij} = \delta_{ji}+\theta_{ji} \tag{7a}$$

$$\therefore \cos(\delta_{ij}+\theta_{ij}) \neq \cos(\delta_{ji}+\theta_{ji})$$

$$\text{Let:} \quad M_{eq\,ij} = \frac{M_{ij}M_{ji}}{M_{ij}+M_{ji}} \quad \text{and} \quad P_{eq\,ij} = \frac{P_{ij}M_{ji} - P_{ji}M_{ij}}{M_{ij}+M_{ji}} \tag{7b}$$

Now, equation (6) can be written,

$$\begin{aligned} M_{eq\,ij} \frac{d^2\delta_{ij}}{dt^2} &= P_{eq\,ij} - V_iV_jY_{ij}\sin\delta_{ij}\sin\theta_{ij} - V_iV_jY_{ij}\Big[\frac{M_{ji}-M_{ij}}{M_{ij}+M_{ji}}\Big]\cos\delta_{ij}\cos\theta_{ij} \\ M_{eq\,ij} \frac{d^2\delta_{ij}}{dt^2} &= P_{eq\,ij} - V_iV_jB_{ij}\sin\delta_{ij} - V_iV_j\Big[\frac{M_{ji}-M_{ij}}{M_{ij}+M_{ji}}\Big]G_{ij}\cos\delta_{ij} \end{aligned} \tag{8}$$

$$\begin{aligned} &\text{Define:} \quad G_{ijeq} = \Big[\frac{M_{ji}-M_{ij}}{M_{ij}+M_{ji}}\Big]G_{ij} \quad \text{and} \quad B_{ijeq} = B_{ij} \\ &\qquad Y_{ijeq} = \big[(G_{ijeq})^2 + (B_{ijeq})^2\big]^{1/2} \quad \text{and} \quad \theta_{ijeq} = \tan^{-1}\Big[\frac{B_{ijeq}}{G_{ijeq}}\Big] \end{aligned} \tag{9}$$

Finally

$$M_{eq\,ij} \frac{d^2\delta_{ij}}{dt^2} = P_{eq\,ij} - V_iV_jY_{ijeq}\cos(\delta_{ij} - \theta_{ijeq}) \tag{10}$$

Equation (10) is in the form which can be used to study the stability with the Equal Area Criteria.

Equation (10) can be described by the One-Machine-Infinite-Bus-Subsystem (1-11) (Fig. 1)

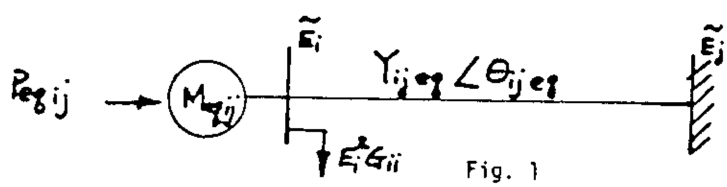

Fig. 1

2.2 Division of Mass

The division of mass may be made imperically according to power transferred and power dissipated for each power line.

$$\frac{M_{ij}}{M_i} = \frac{V_i V_j Y_{ij}^{PR}\cos(\delta_{ij}^{PR}-\theta_{ij}^{PR})}{\sum\limits_{\substack{k=1\\k\neq i}}^{n} V_i V_k Y_{ik}^{PR}\cos(\delta_{ik}^{PR}-\theta_{ik}^{PR})} \;;\; i,j=1,\cdots n,\; i\neq j \tag{11}$$

2.3 Division of Acceleration Power

$$\frac{P_{ij} - V_i V_j Y_{ij}\cos(\delta_{ij}-\theta_{ij})}{M_{ij}} = \frac{P_i - \sum\limits_{j=1}^{n} V_i V_j Y_{ij}\cos(\delta_{ij}-\theta_{ij})}{M_i} \tag{12}$$

and

$$P_{ij} = \frac{M_{ij}}{M_i}\Big[P_i - \sum_{\substack{j=1\\j\neq i}}^{n} V_i V_j Y_{ij}\cos(\delta_{ij}-\theta_{ij})\Big] + V_i V_j Y_{ij}\cos(\delta_{ij}-\theta_{ij})$$

3. Example (Four-Machine Power System)

3.1 Formulation

Equations of motion for all Four-Machines are written as follows:

$$M_i \frac{d^2\delta_i}{dt^2} = P_{mi} - \sum_{\substack{j=1\\j\neq i}}^{4} \frac{E_i E_j}{x_{ij}} \sin\delta_{ij} \quad ; \; i=1,\cdots 4 \tag{13}$$

Assuming the pre- and post-fault systems are same,

$$M_i \frac{d^2\delta_i}{dt^2} = \sum_{\substack{j=1\\j\neq i}}^{4} \frac{E_i E_j}{x_{ij}}(\sin\delta_{ij}^{o} - \sin\delta_{ij}) \;;\; i=1,\cdots 4 \tag{14}$$

Rewrite equation (14)

$$\Big(\sum_{\substack{j=1\\j\neq i}}^{4} M_{ij}\Big)\frac{d^2\delta_i}{dt^2} = \sum_{\substack{j=1\\j\neq i}}^{4} \frac{E_i E_j}{x_{ij}}(\sin\delta_{ij}^{o} - \sin\delta_{ij}) \;;\; i=1,\cdots 4 \tag{15}$$

where

$$M_i = \sum_{\substack{j=1\\j\neq i}}^{4} M_{ij} \tag{16}$$

From Equation 15, $\frac{4(4-1)}{2}$ = 6 Subsystems are defined

$$\frac{d^2\delta_{ij}}{dt^2} = \frac{1}{M_{ij}}\frac{E_i E_j}{x_{ij}}(\sin\delta_{ij}^{o} - \sin\delta_{ij}) - \frac{1}{M_{ji}}\frac{E_j E_i}{x_{ji}}(\sin\delta_{ji}^{o} - \sin\delta_{ji}) \tag{17}$$

$$= \left[\frac{M_{ji}\left(\frac{E_i E_j}{x_{ij}}\right) + M_{ij}\left(\frac{E_j E_i}{x_{ji}}\right)}{M_{ij}\, M_{ji}}\right](\sin\delta_{ij}^{o} - \sin\delta_{ij}) \;;\; \begin{matrix} i=1,\cdots 3 \\ j=i+1,\cdots 4\end{matrix} \tag{18}$$

Note: $x_{ij} = x_{ji}$; $P_{eq\,ij} = \frac{E_i E_j}{x_{ij}}\sin\delta_{ij}^{o}$ and $M_{eq\,ij} = \frac{M_{ij}M_{ji}}{M_{ij}+M_{ji}}$

Equation 17 can be re-written as

$$M_{eq\,ij}\frac{d^2\delta_{ij}}{dt^2} = P_{eq\,ij} - \frac{E_i E_j}{x_{ij}}\sin\delta_{ij} \;;\; \begin{matrix} i=1,\cdots 3 \\ j=i+1,\cdots 4\end{matrix} \tag{19}$$

$$\text{Define} \quad E_{ij} \triangleq E_i E_j \;,\; E_\infty \triangleq 1 \;\text{ and }\; \angle E_\infty \triangleq 0 \tag{20}$$

Equation 19 is now written as

$$M_{eq\,ij}\frac{d^2\delta_{ij}}{dt^2} = P_{eq\,ij} - \frac{E_{ij}\,E_\infty}{x_{ij}}\sin\delta_{ij} \;;\; \begin{matrix} i=1,\cdots 3 \\ j=i+1,\cdots 4\end{matrix} \tag{21}$$

3.2 Stability Criteria

The System Stability can be assessed by applying the equal area criteria (1-11) to each subsystem (equation 21). The System is stable if, and only if, all subsystems are stable.

4. Conclusion

The stability problem of a large scale power system is considered. The method presented is very useful in the applications to practical power system stability analyses. It allows us to find decoupled stability criterion which requires only a local monitory system.

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