Wave Propagation & Information
Chapter 4

Field Theory

Current flow through a conductive substance creates an electromagnetic field around the substance as shown in Figure 4-1. The field is made up of mutually perpendicular electric and magnetic waves of varying strengths (intensities) called E fields and H fields respectively. These field strengths are vector quantities, having both magnitude and direction. Neither field can exist independently: whenever there exists a magnetic field, an electric field also exists. Radio waves are defined as moving electromagnetic fields with E and H field components at right angles to each other, and having velocity in the direction of travel.

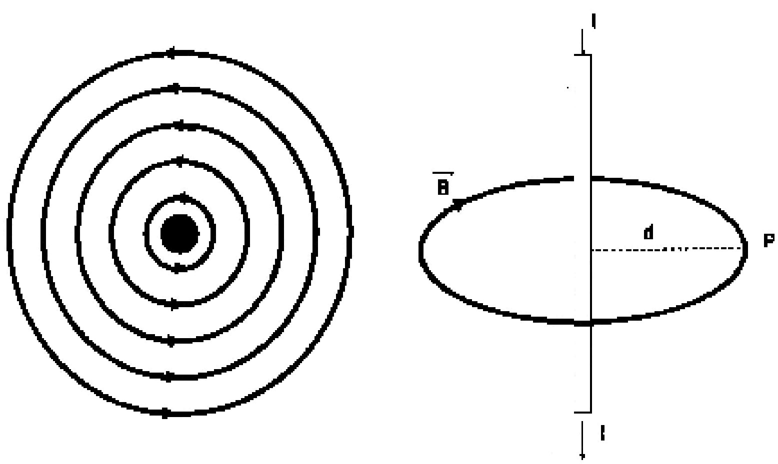


Figure 4-1 Fields & Current Flow Around a Conductor

If the current flowing through the conductor is a sinusoidal varying or pulsed current, the electromagnetic field around the conductor will expand and contract, and a portion of the energy stored in the field will be radiated away from the current path. The greater the low frequency component of current flow, the greater the magnetic field strength.

As the E-field expands and collapses, completely closed loops of E-flux lines are formed and "pushed" into space by subsequent expanding flux lines. The radiated E-field generates its own accompanying H-field since there can be no conductive current in free space, the radiated E-field acts as a displacement current which generates the H-field. Figure 4-2 and 4-3 show the resulting fields associated with a loop and a wire source. Individual field components can be calculated from the equations of Figure 4-4. Figure 4-5 shows the components of an electromagnetic wave including direction of travel and orientation.

$$for\ \ r \ll \frac{\lambda}{2\pi} = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi = 377\ \Omega$$

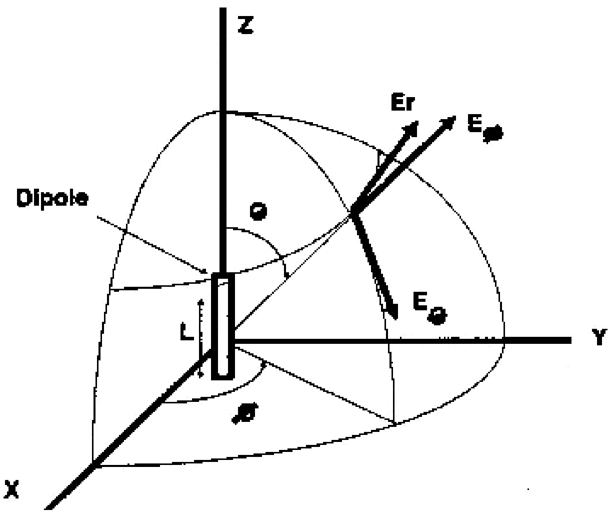


Figure 4-2 Wire Source

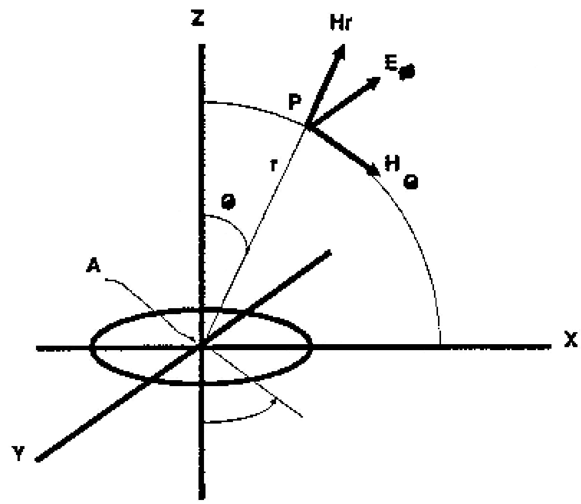


Figure 4-3 Loop Source

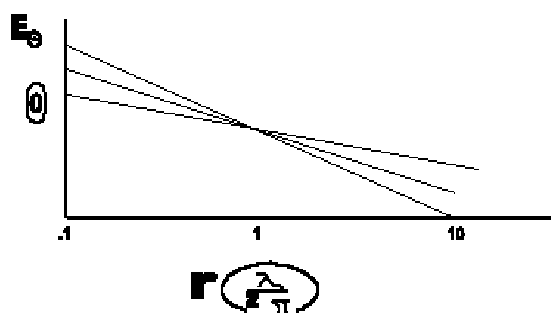


Figure 4-4 For Field Component Calculations

$$H_{\phi} = \frac{ID\pi \sin\theta}{\lambda^2} \left[\left(\frac{\lambda}{2\pi r}\right)^2 \sin\psi + \left(\frac{\lambda}{2\pi r}\right) \cos\psi \right]$$

$$E_{\theta} = \frac{Z_o ID\pi \sin\theta}{\lambda^2} \left[-\left(\frac{\lambda}{2\pi r}\right)^3 \cos\psi - \left(\frac{\lambda}{2\pi r}\right)^2 \sin\psi + \left(\frac{\lambda}{2\pi r}\right) \cos\psi \right]$$

$$E_r = \frac{2 Z_o I D \pi \cos\theta}{\lambda^2} \left[\left(\frac{\lambda}{2\pi r}\right)^3 \cos\psi + \left(\frac{\lambda}{2\pi r}\right)^2 \sin\psi \right]$$

where

where	Z_o	= free space impedance
	I	= current in short wire (doublet)
	D	= length of short wire doublet in which D << λ
	θ	= zenith angle to radial distance r
	λ	= wavelength corresponding to frequency, f = c/λ
	r	= distance from short wire doublet to measuring or observation point
	Ψ	= 2πr/λ - ωt
	t	= time = 1/f
	c	= 3×10^8 m/sec
	ω	= radial frequency = 2πf

The fluctuating energy produces an induction field associated with the stored energy, and a radiation field. The static and induction fields are called "near" fields, and the radiation field is called the "far" field. Figure 4-6 describes the near and far field relationship in terms of the transition point. Figure 4-7 describes individual fields in terms of wave impedance. The static field varies inversely as the third power of the distance from the antenna, so that it becomes negligible at distances greater than about one one-hundredth (1/100) of a wavelength from the radiating source.

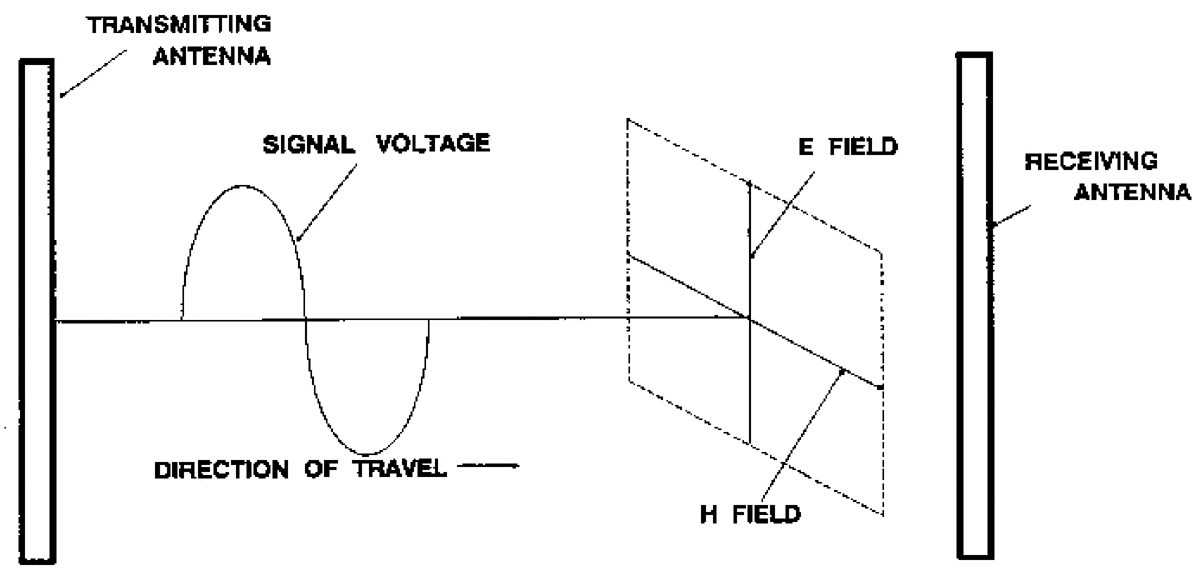


Figure 4-5 Components of an Electromagnetic Wave

Power Transmission

Standard propagation relates to delivering power to a conductive object which in turn, based on the object's loading with respect to free space impedance, delivers this power through the expanding and contracting fields to another conductive object located at some other point. For power transmission, antennas are used to match the impedance of the transmission source to that of free space.

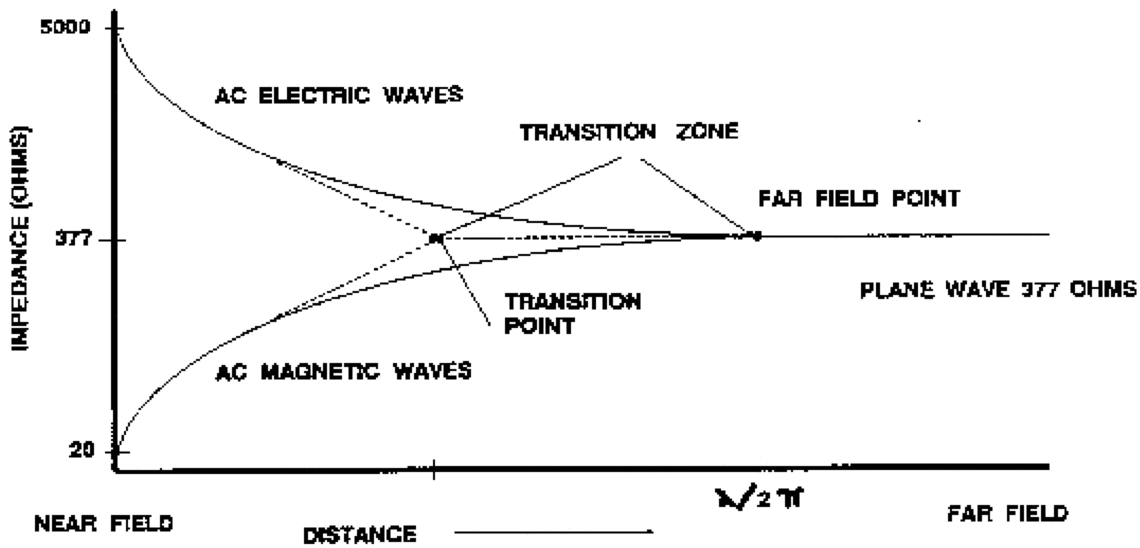


Figure 4-6 Near and Far Field Conditions

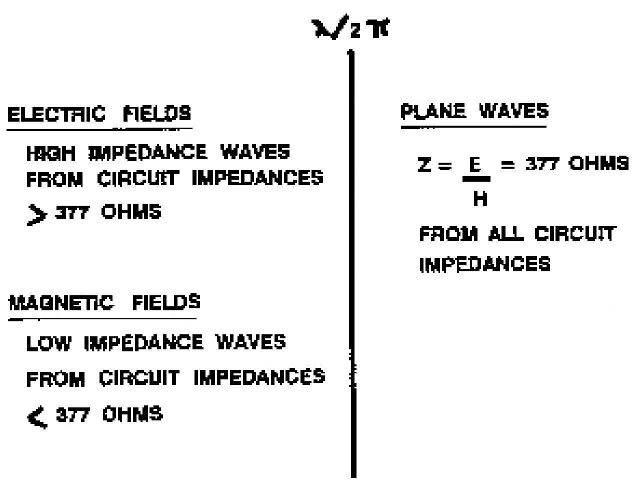


Figure 4-7 Near and Far Field Impedance Relationships

IMPEDANCE AND POLARIZATION

Propagation is based on the ability to deliver power to an object that is loaded with respect to free space impedance.

For power transmission, antennas are used to match the impedance of the transmission source to that of free space.

Polarization is determined by the direction of the lines of force that make up the E-field.

Maximum antenna pickup results when the antenna is oriented in the same direction as the radiated field.

Polarization of a radiated wave is determined by the direction of the lines of force that make up the E-field. Maximum antenna pickup results when the antenna used is oriented in the same direction as the E-field components. To evaluate power transfer, it is convenient to use, as a standard antenna, one that has a length that is small compared to the wavelength, designated as a doublet. Doublets can be used for both transmitting and receiving. In free space, optimum transmission is achieved when the two doublets are parallel to each other and perpendicular to the line connecting their centers. If their distance apart, d, is large compared to the wavelength, the ratio of power transmitted to maximum useful power received is found from:

$$\frac{P_2}{P_1} = G_1 G_2 \left[\frac{3\lambda}{8\pi d}\right]^2$$

P_2 (P_r) is the power delivered to a matched load at the output terminal of the receiver, and P_1 (P_t) is the power fed to the transmitting antenna. Gain for a directive antenna is the ratio of the power transmitted by a doublet to the power transmitted by the antenna to produce the same response in a distant receiver when both antennas are adjusted for maximum power transfer. Gain can be expressed in relation to a doublet, or relative to a hypothetical isotropic radiator, a lossless radiation source with the same power density in all directions. An isotropic radiator is shown in Figure 4-8.

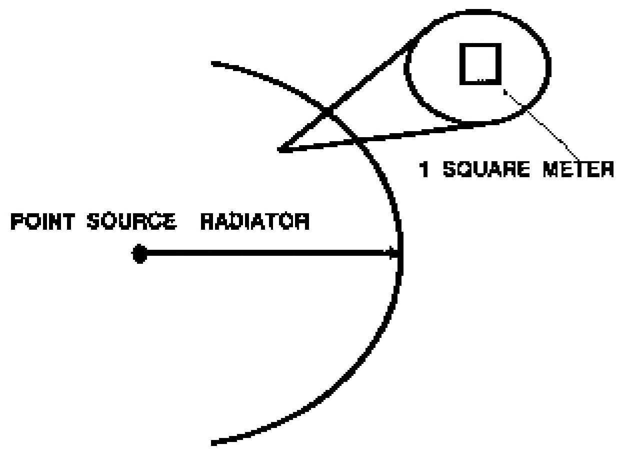


Figure 4-8 An Isotropic Radiator

If transmission takes place over a conductive body (ground) or in a refracting atmosphere, etc., the power ratio is expressed as:

$$\frac{P_2}{P_1} = G_1 G_2 \left[\frac{3\lambda}{8\pi d}\right]^2 A_p^{\;2}$$

In this case, G_1 and G_2 are the antenna gains of the transmitting and receiving systems, and A_p is the "path factor" relating to the physical characteristics of the transmission path. A nomogram for free space transmission when $G_1 = G_2 = A_p = 1$ is provided at the end of this paper.

John Pierce developed the following equation for path loss over the ground with no structures between transmitter and receiver.

$$\mathbf{P_r = P_t + G_t + G_r + [-32.45\ dB - 20 \log d - 20 \log f]}$$

P_r = received power (dB) relative to 1 Watt
P_t = transmitted power (dB) relative to 1 Watt
G_t = transmitting antenna gain over isotropic in dB
G_r = receiving antenna gain over isotropic in dB
d = distance (km) between transmitting and receiving antennas
f = frequency in MHz

The part of the equation between the brackets is the actual path loss. The 32.45 dB is a constant that relates the distance in km and frequency in MHz.

Isotropic Radiator Example

Assume a perfect isotropic radiator is driven with 1 watt of power. Determine how much power is present at a distance of km away. Figure 4-9 shows Figure 4-8 modified to describe the problem solution: If the space between the radiator and the surface of the imaginary sphere is a vacuum or clear air, all the power radiated (1 watt) will impinge on the sphere's inner surface uniformly. The expression for the surface area of a sphere is:

$$Area\,(A) = 4\pi\, R^2$$

In the example, A = 4π x 1000 x 1000 = 12,566,368 sq. meters.

1 watt from an isotropic radiator produces 1/1257.3667 =
7.96×10^{-8} watts per square meter at 1 km, or - 70.99 dBW/m.

A sphere with a radius of 2 km would have four times as many cubic meters of surface area, and only one-fourth the power per square meter. Power density falls of inversely as the square of distance.

The effective area of an isotropic receiving antenna is equal to the wavelength squared divided by four pi. Since frequency is inversely proportional to wavelength, power received by an isotropic antenna is inversely proportional to the square of frequency (-20 log f). Figure 4-10 described an isotropic receiving antenna.

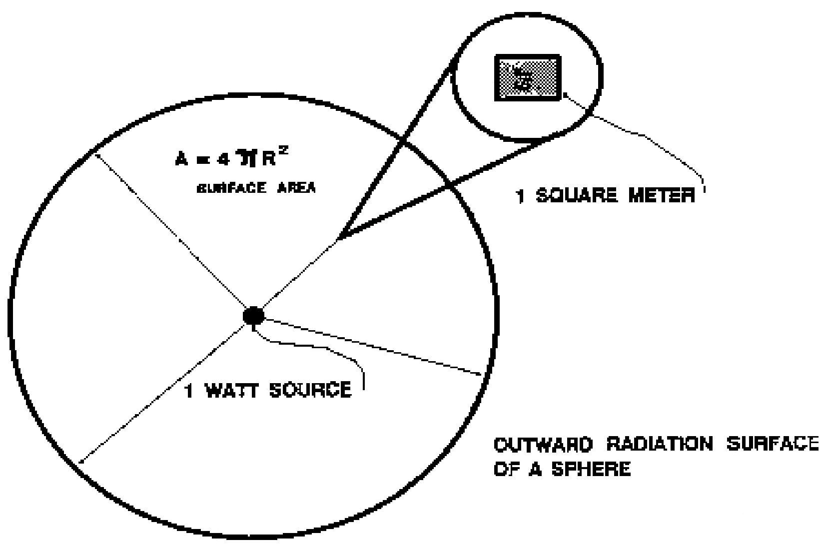


Figure 4-9 Perfect isotropic Radiator Driven With 1 Watt

To use the expression for power density at the receiving site equal to electric field strength squared divided by the impedance of free space, the free space impedance must be calculated from:

$$Z_v = \sqrt{\frac{\mu}{e}}$$

$\mu = 4\pi \times 10^{-7}$ Henrys per meter = 1.257×10^{-6} H/m

$e = \frac{1 \times 10^{-9}}{36\pi}$ Farads per meter

$= 8.842 \times 10^{-12}$ F/m

This value is approximately 377 (376.7) ohms. In a sense, an antenna can be considered to be a transformer, transforming the impedance of free space to the impedance of its output terminals.

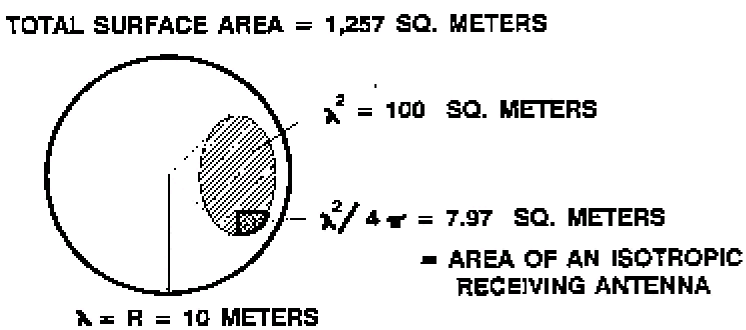


Figure 4-10 Isotropic Receiving Antenna

The electric field strength at the receiver is given by:

$$E = \frac{3\sqrt{5}}{d} \sqrt{P_1 G_1}\, A_p$$

where E is in volts per meter, and Pt is in watts. If E is known, the power delivered by the receiving antenna to a matched load is:

$$P_2 = \frac{E^2}{120\pi} \frac{3\lambda^2}{8\pi} G_2$$

For the isotropic radiator fed with one watt, the field strength at 1 km would be:

$$E = \sqrt{PD * 120 \pi}$$

$$E = \sqrt{7.96 \times 10^{-8} \times 376.7} = 5.48 \text{ mV/m}$$

A graph of field strength as a function of distance to transmitter os included at the end of this paper.

Wave Propagation Conclusions

Energy propagates into space based on the impedance mismatch between the transmission line source and the load on the line. If the transmission line is only slightly mismatched, as is the case with the majority of sensitive emission problems encountered, only small higher frequency signals will propagate into space. However, if the mismatch is large, such as unterminated, or if the wire is terminated by an antenna, then significantly more energy radiates into the outside world.

Oscillations at Transmission Line Interfaces[1]

Figure 4-11 is a transmission line of 1000 ohms surge impedance mating with a short length of line of surge impedance of 200 ohms. This in turn is mated with a continuation of the 1000 ohm transmission line. The time required for a wave to travel the length of the 200 ohm line is Δt. Coming into the junction from the left is a current wave of amplitude 1.0. At this junction the transmission coefficient is $\gamma = 1.667$ and the reflection coefficient is $\delta = 0.667$. Accordingly, a transmitted wave of 1.667 amplitude enters the 200-ohm line and a reflected wave is launched from right to left of amplitude 0.667. On either side of the vertical separation lines, the voltage must be equal. The incident voltage (to the left of the separation line) is equal to the sum of the incident and reflected components for 1.667 amplitude. The wave that is transmitted into the 200 ohm section of line then propagates

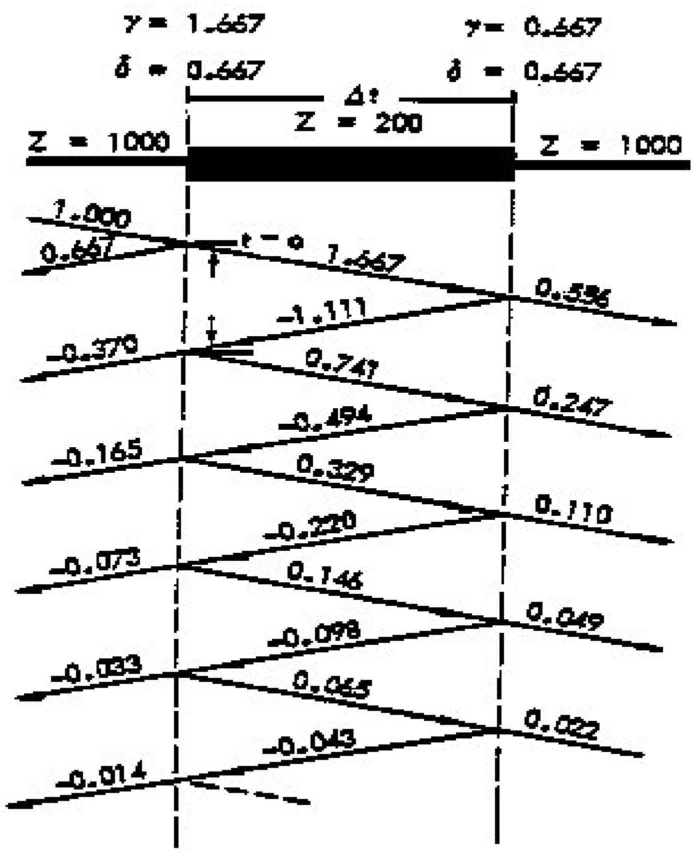


Figure 4-11 Three Section Transmission Line Showing Amplitude of Reflections

[1] *The majority of this section derived from General Electric Aircraft Lightning Protection Note 75-1.*

to the right to the next transition point where a reflection of amplitude -1.111 occurs, and a transmitted wave of amplitude 0.556 is propagated to the right and out of the paper.

The first reflection traveling right to left then meets the first transition point at a time 2Δt after the incident wave hit the first junction. At that time, there is generated a second reflection traveling right to left of amplitude 0.741 and a transmitted wave going right to left of 0.370. At any time, the magnitude of the wave at the first transition is the sum of all of the wave components on either side of the first vertical demarcation line. Accordingly, at time 2Δt the amplitude at the first transition point is 1.0 + 0.667 - 0.370 or 0.1297. The amplitude at any other point and at any other time is likewise the sum of all the transmitted and reflected wave components above that point at that time. For instance, at the midpoint of the 200-ohm line, the current is zero until a time 1Δt. This time, the amplitude jumps to 1.667 and remains there until a time 3Δt, when the amplitude becomes 1.667 - 1.111 or 0.556. This bookkeeping process of keeping track of the transmitted and reflected wave components may be continued as long as necessary.

The total pattern of development of these waves for the conditions of Figure 1 (and assuming Δt equals 0.1 microseconds) is shown on Figure 4-12. The voltage at the input of the 200-ohm line is seen to rise to an amplitude of 0.167 and decay in a series of steps reaching essentially its final amplitude after about 1 microsecond. Current does not begin to come out of the line until 0.1 microsecond, at which time it begins to jump to its final value in a series of steps. The current at the midpoint of the 260 ohm line oscillates bark and forth with a period 2Δt = 0.2 microseconds, or a frequency of 5 MHz.

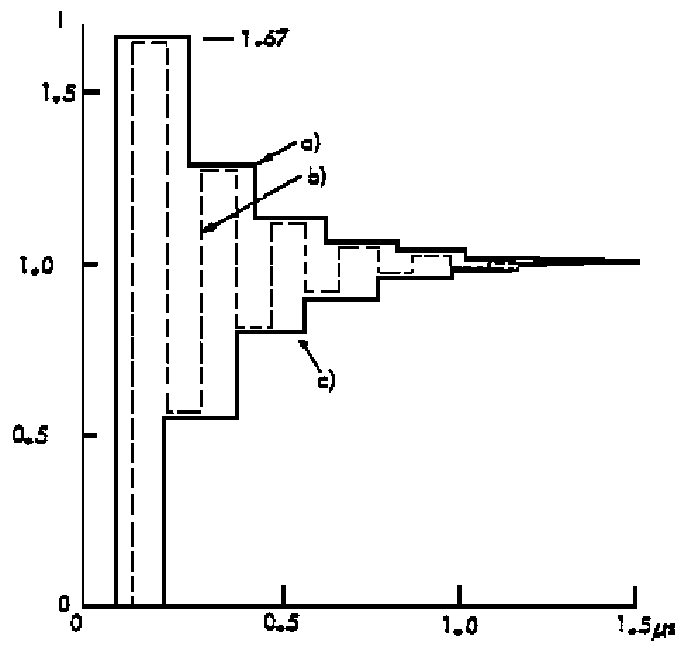


Figure 4-12 Response to Step Function Current

A) Input Current

B) Current at Center of 200 ohm Line

C) Output Current

Figure 4-13 shows types of voltage and current distribution produced on a transmission line by different load impedances for the case of a transmission line having low attenuation and characteristic impedance that is resistive.

Information Theory

There is a considerable amount of confusion existing in the TEMPEST industry related to information theory, information ratios, and the application of ratios to practical analysis problems. This section attempts to help clarify the confusion in an unclassified format by explaining the requisite background information regarding the theory before developing an unclassified figure of merit (an IR) for breaking down detected signals into useful information. Other figure of merits may have been developed for specific purposed, but the information and techniques presented herein are generic, and will help in the understanding of other similar values.

Information is a quantitative term, measured by the degree to which is clarifies that which is unknown; a totally predictable event contains no information. In general, information has the property of reducing the uncertainty of a situation. The uncertainty is called entropy (H), and entropy exists to the extent that information is lacking (Information + Entropy = 100% or I + E = 1). If the entropy of a situation is small, only a small amount of information is required to clarify it. If the entropy is large, then much more information will be required before the uncertainty can be replaced by an acceptable degree of clarity.

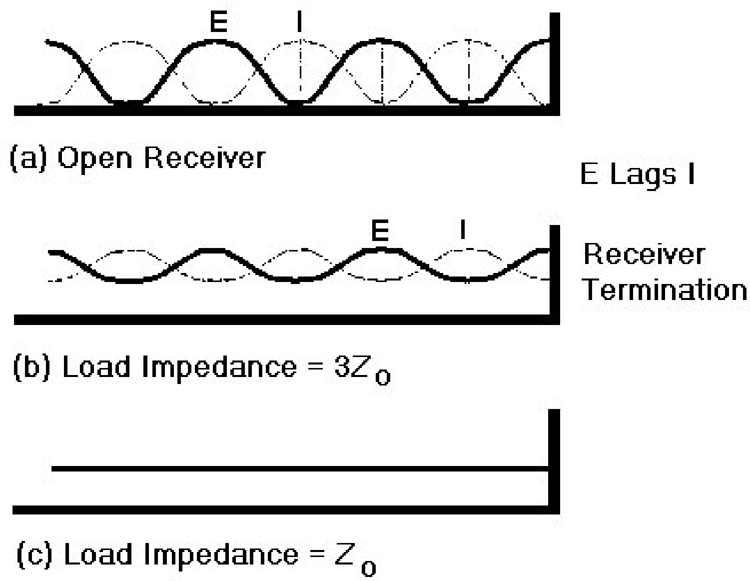


Figure 4-13 Effects of Transmission Line Loading

If the probability of an event E occurring is P(E), then the information obtained

when E occurs is:

$$I(E) = \log \frac{1}{P(E)}$$

with the choice of base for the logarithm corresponding to the unit of information (i.e. base 2 for bits, base 10 for Hartleys), where 1 Hartley is equal to 3.22 bits. In this case, there is a clear distinction between a bit as a unit of information or a binary number, and a baud as a unit of signaling speed.

Baud is characterized by the presence or absence of a pulse in a channel, and is a measure of the maximum rate of pulses (code elements) per second in the system. Baud rate is found by taking the reciprocal of the length (in seconds) of the shortest pulse used in creating a character. As an example, the length of a pulse of baudot code as used with 60 wpm teletypewriters is 0.022 seconds (22 ms). The baud rate is the reciprocal (1/0.22 =45.45 bauds), which is the highest possible baud rate of this signal. Figure 1 describes the condition. In TEMPEST work, bauds are not considered.

Bit is the abbreviation for binary digit, with both binary states called bits, since both states carry the same amount of information. The number of bits required to identify any particular selection from a group of N possible selections is:

$$I = \log_2 N$$

provided that all N selections have equal opportunity of being chosen.

The term "byte" is used to describe a group of consecutive bits that are treated as a unit. Most computers are designed to use byte-sized characters of eight bits. The transmission of both bits and characters are usually measured in terms of so many per second, whereas words are measured in so many per minute.

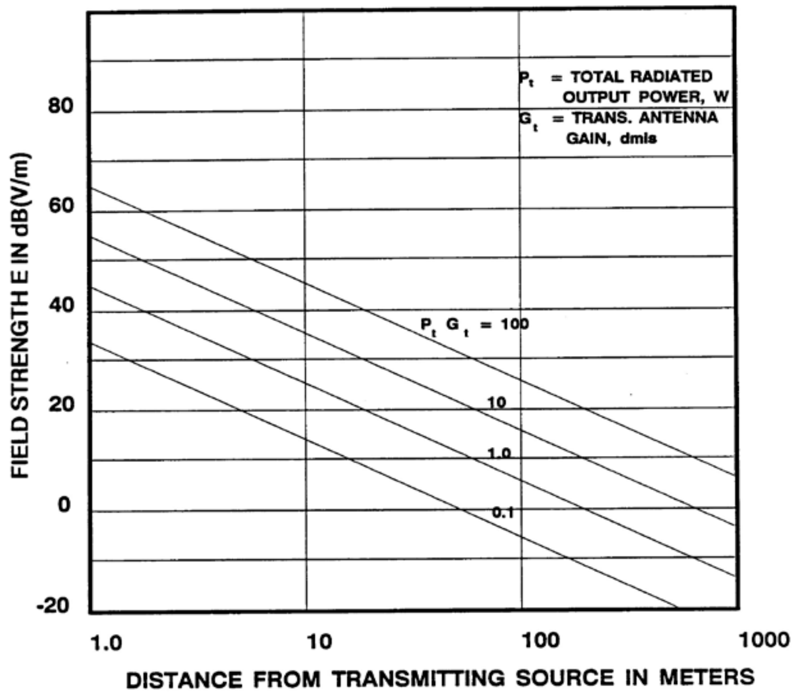


Figure 4-14 Free Space Loss with Distance

Data is transferred across communications channels using various baseband coding techniques, depending on the number of lines available and the application..

Messages

A message, in information theory, is merely the output of some information source. The quantitative value of a message is based on several factors. First, it must be established how much was known about the contents of the message before it was received. Second, it must be known how many messages were in the set from which the message was chosen. Third, to be precise, there should be some way of knowing the probability of each event that the message could describe.

A few examples may help clarify what is being described. If the source is a telephone transmitter, the message would be the analog voltages impressed on the telephone line. If the source is a teletypewriter, the message could be a character, one of the bits making up a character, or the message could be the entire word. The composition of a message can therefore be a number of things depending on how it is defined.

A further breakdown relates to language. If the message is in commonly spoken English, the message consists of a reduced set of words, each made up of letters of the English alphabet. If the message is babilities possible.

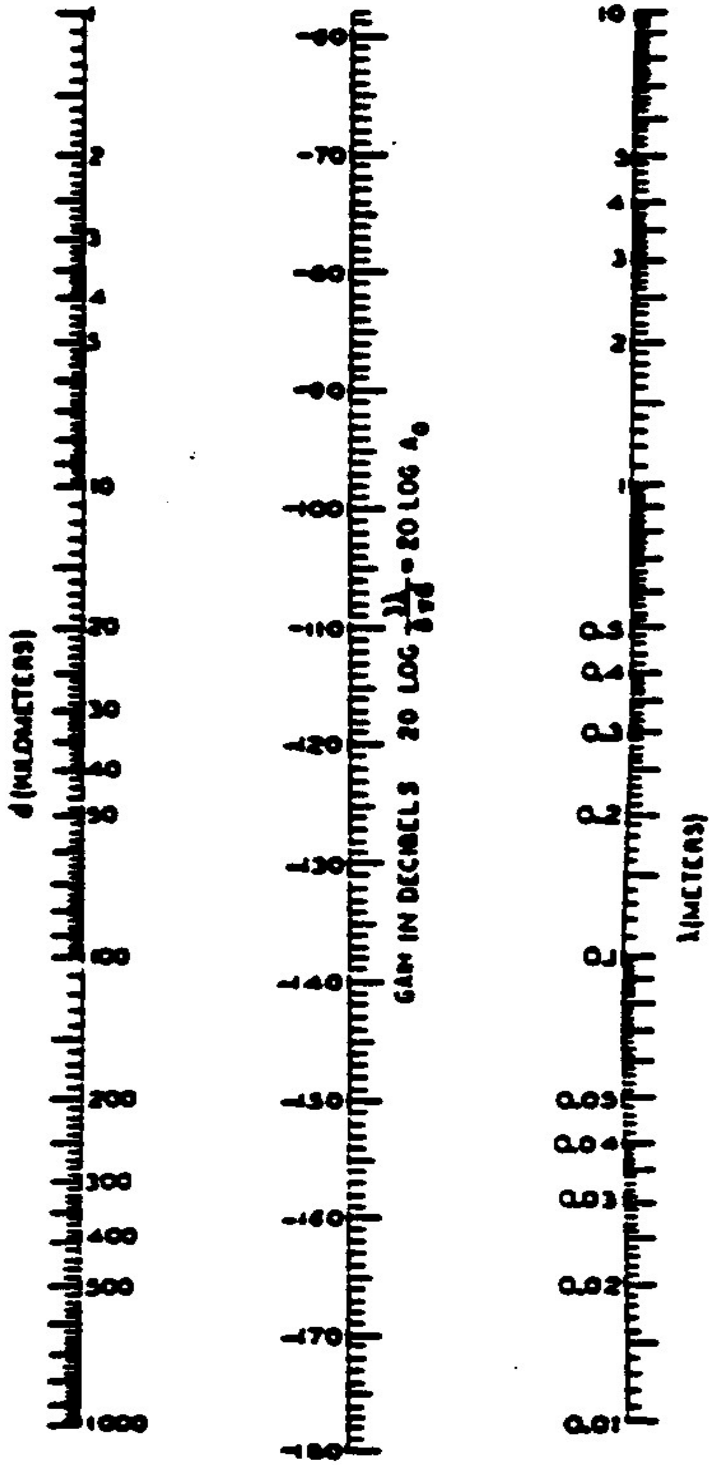


Figure 4-15 Nomogram for Free Space Transmission Between Parallel Doublets

Assuming that the probability of occurrence of a symbol in the source language does not depend on the symbol(s) preceding it (a definition of a zero-memory source), each symbol gives information equal to:

$$I(s_i) = \log \frac{1}{P(s_i)}$$

and the average information per symbol is:

$$H(S) = \Sigma\, P(s_i)\, I(s_i)$$ H(S) is the entropy of the source

Obviously, a zero-memory source is not realistic in a communications context. In English, the probability of a "h" occurring is greater is the preceding symbol is a "t" rather than an "l". Therefore, the sources considered here will be ergodic Markov sources, sources where the occurrence of a symbol depends on a finite number (the order of the Markov source) of preceding symbols, and where if the source input is long enough, a typical sequence will occur.

From various empirical sources, the entropy of English (26 letters and a space) is:

(1) As a zero memory source	4.03 bits/symbol
(2) As a first order Markov source	3.32 bits/symbol
(3) As a third order Markov source	3.10 bits/symbol

Shannon, the father of information theory, has by theoretical methods estimated the entropy of actual English to be between .06 and 1.3 bits/symbol.

Codes

Regardless of code used (ASCII or even COMSEC), the intent of reducing uncertainty is the decoding a of symbol sequence. Even though English is a code for thought, the information above is not adequate to vigorously describe an encoded information channel. In this regard, a better description of code requirements is necessary.

The codes we will be considering are Instantaneous, Uniquely Decodable, and Non-Singular Black Code. These codes are described below.

- Block Code - A code where every source is represented by a finite fixed sequence of code symbols, called a code word.
- Non-Singular - All code words are unique.

- Uniquely Decodable - For every finite sequence of the source alphabet, the encoded sequence is a Block Code. For example, binary numbers with leading zero suppressed are not uniquely decodable (100 could be 4 or 1,0,0 or 2,0).
- Instantaneous - Can be decoded without decoding succeeding sequences.

Without going into an involved proof, it is intuitive to state that the longer the code required for a source, the less information per code symbol. The mathematics involved to this point is nice, but much too theoretical to be of use to a TEMPEST engineer. We need to direct our discussions to a more realistic realm, the definition of an information channel. An information channel consists of an input alphabet A = {a(i)} where i = 1,2, ...r, an output alphabet B = {b(i)}i where i = 1, 2, ...r, and a set of conditional probabilities p[b(j)/a(i)] such that b(j) will be received in a(i) is sent for all j and i. This allows the introduction of noise into a communication.

An Information Channel and Shannon Capacity

Let's first look at channel noise power. As signal and noise approach the same power level, with constant channel bandwidth, the signal must exist for longer periods of time in each discrete state in order to be detected. The theoretical maximum bit rate C, through a channel of bandwidth (BW) and signal-to-random-noise power ratio S/N is:

$$C = BW \, Log_2 (1 + S/N)$$

The S/N power ratio indicates the relative strength of the signal to that of channel noise.

$$Log_2 \, X = Log_2 \, 10 \, Log_{10} \, X$$

$$Log_2 \, 10 = 3.32193 = 1/Log_{10} \, 2$$

In the presence of noise, a binary signal is obviously more easily detected than one using several bits per code element. The required minimum S/N power ratio from a known bit rate and bandwidth is:

C/BW or S/N=2

where C is the maximum bit rate through a channel.

As the bit content (number of levels) of a code element is increased, a corresponding increase in the S/N power ratio must be made to maintain equal detection capability relative to a binary signal. For a common binary (digital) channel, S/N = 3, or signals 4.8 dB above the noise level are directly detectable.

Noise uncertainty leads also to effects on probability calculations. Using arguments parallel to those for zero- memory and Markov sources, we can arrive at the channel equivocation:

$$\frac{\underline{Ln/n}}{\lim n\rightarrow\infty} = H(A/B)$$

It must be stated that channel equivocation applies only to instantaneous uniquely decodable codes. Channel equivocation is the average knowledge gained from observing an output symbol in B produced by an input symbol for A. Conversely, H(A/B) bits are required to produce an output symbol in B from an input symbol in A. Since it takes H(A) bits to define an input symbol in A, the average information gained from receiving a symbol in B is:

$I(A,B) = H(A) - H(A/B)$ the mutual information channel.

The mutual information of a channel contains less information than the source because of the noise uncertainties. This decrease in information from the observed signal reduces the efficiency of communication transmission in a manner similar to code efficiency.

$$\frac{\text{Channel information}}{\text{source code information}} = \frac{H(A) - H(A/B)}{H(A)}$$

The transmission efficiency is less than 1, and mitigates against communication. If a redundancy is built into the source code, either by some error checking protocol, or by semantics, as in the case of natural languages, communicability factor affects efficiency. This communicability factor is predicated upon observing the output signal. If an unobserved channel is postulated, as would be the case in encryption schemes, communicability becomes:

$$K\left\{\frac{l}{L-H(A)}\right\}\left\{\frac{H(A) - H(A/B)}{H(A)}\right\} - H(K)$$

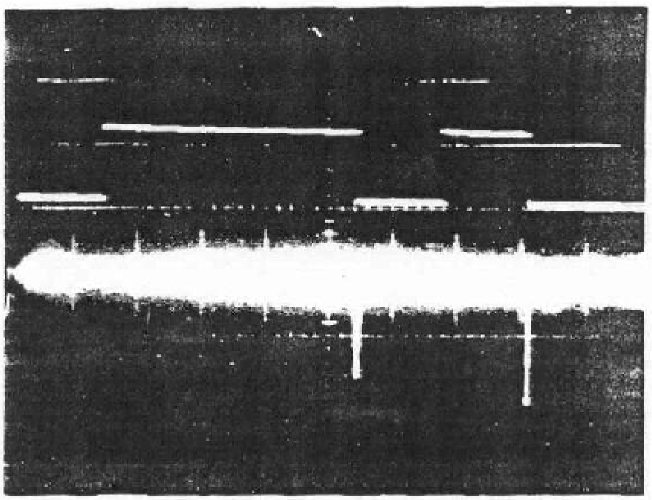
The Gabrielson/ Caldwell IR (GCIR)

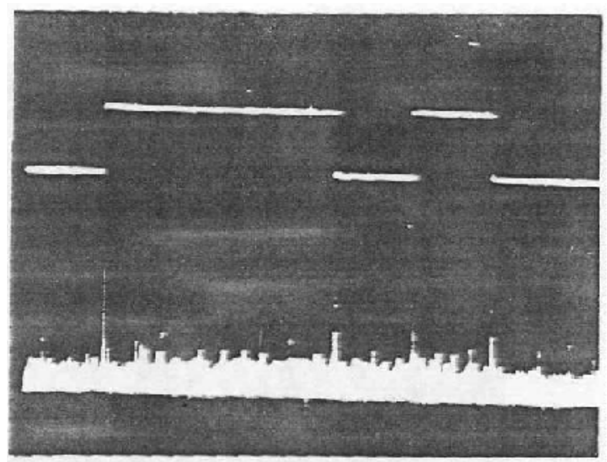
and can be reduced to any level. The above equation can be considered an information ratio figure of merit for information detectability. Negative values of IR would indicate a "red herring" strategy. Cascaded channel cases have not been addressed for the sake of simplicity of presentation. Also, since mutual information is additive, signals can be expressed directly by the above communicability factor.

What all this theory means is that the use of information ratios, IR's, provides a statistical measure of the information content of a recovered signal. One important factor is the redundancy of the language being used. English is approximately 75% redundant. Other factors include the probability that certain characters are part of the massage. Rigorously applying an IR to a practical problem is not an easy task, since a solution involves the use of a much higher level of mathematics than used thus far in this derivation. The basic intent was to show that information might exists in many formats, coded emissions being but one such media. The mere detection of a coded signal in a media is not sufficient to determine if enough information is present to identify the message being sent.

The TEMPEST Signal

By putting communications theory, modulation, and propagation together with IRs, it is now possible to understand how a non-traditional carrier signal can be modulated by a signal bearing source and become a signal that can be subsequently detected through radiated or conducted means. The carriers can be conducted in the ground system, or can be generated by an emission source like an oscillator or other machine cycle related spurious emission. The signal bearing source is generally directly related to some circuit or discrete circuit element that is processing a sensitive data or voice signal. By correctly interpreting what is detected, it is potentially possible to determine the source data for a particular transmission. The following figures from NACSEM 5000 TEMPEST Fundamentals are typical of detected information bearing signals.





References

Two other additional figures are provided for this chapter plus the two references below are key to understanding propagation.

Burrows, C.R., and Attwood, S.S., Radio wave propagation, Academic Press, Dec, 1948.

Tendick, C., Understanding Freespace Propagation, MSN & CT Magazine, October 1987.